

Coherent macroscopic quantum tunneling in boson-fermion mixtures

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We show that the cold atom systems of simultaneously trapped Bose-Einstein condensates (BEC's) and quantum degenerate fermionic atoms provide promising laboratories for the study of macroscopic quantum tunneling. Our theoretical studies reveal that the spatial extent of a small trapped BEC immersed in a Fermi sea can tunnel and coherently oscillate between the values of the separated and mixed configurations (the phases of the phase separation transition of BEC-fermion systems). We evaluate the period, amplitude and dissipation rate for ^{23}Na and ^{40}K -atoms and we discuss the experimental prospects for observing this phenomenon.

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The tunneling of a macroscopic (or collective) variable of a many-body system through a classically forbidden region, macroscopic quantum tunneling (MQT), is a phenomenon of fundamental interest [1] and a recurring theme in a variety of fields ranging from nuclear (fission) and condensed matter physics (e.g. quantum magnets [2], SQUIDS) to quantum optics (macroscopic Schrodinger cat states [3] and beyond-standard limit measurements). Nevertheless, stringent tests under well-understood and controlled conditions remain an experimental challenge. Cold atom gases, arguably the cleanest and best understood mesoscopic systems which, furthermore, offer unprecedented control knobs such as the ability to vary the inter-particle interactions [4], now provide an intriguing candidate laboratory for the study of MQT.

The first cold atom MQT proposals [5] suggested observing the collapse of a trapped dilute gas Bose-Einstein condensate (BEC) of mutually attracting bosons. However, the experimental results [6, 7] were either too sensitive to particle number to distinguish MQT from classical collapse [6], or the analysis was complicated by more complex dynamics (such as 'clumping') [7]. Evidence of coherence (of the many-body system taking on a linear superposition of states that correspond to the macroscopic variable residing on either side of the barrier) is even more difficult to gather. Such coherence would be more readily observable in the MQT between *long-lived* states, in which case one could set up a coherent population oscillation between the many-body states. Such long-lived states naturally occur in (zero-temperature) first order phase transitions in which the order parameter, which provides the macroscopic variable, can tunnel through the barrier of its Landau-Ginzburg potential. In the infinite system limit, the coupling between the two states rigorously vanishes, but finite-size cold atom systems of moderate particle numbers provide, once again, a promising candidate to observe the MQT coherence between states of different phases, as we show below.

An earlier proposal to observe MQT between states in which the components of a BEC-mixture arrange themselves differently in space, involved a very low coupling on account of the small spatial overlap between the single component densities in the different states [8]. In

this paper, we propose that MQT can be realized and its coherence, perhaps, observed in trapped gas mixtures of a single-component fermion system and a BEC. Such mixtures are currently created [9] e.g. in the sympathetic cooling scheme in which the colder BEC cools the fermions. The tunneling and coherent oscillations that we target would occur between states of the mixed and separated phases in the phase separation transition of the fermion-BEC mixture [10]. Such transitions could be accessed by varying the scattering length of the boson-fermion interaction [11].

We consider N_B atomic bosons confined in a spherically symmetric harmonic trap (of frequency ω_T) interacting with a much larger system of atomic fermions. For simplicity we assume the fermions to occupy an infinite volume. The Hamiltonian of the bosons is described by the standard Gross-Pitaevskii (GP) form [1], i.e., with inter-particle interactions described by a contact potential ($\propto \lambda_{BB}\delta(\mathbf{r} - \mathbf{r}')$), which we choose to be repulsive ($\lambda_{BB} > 0$). We assume that the interaction of bosons with fermions is also contact-like, contributing $\lambda_{BF}|\Psi_B|^2|\Psi_F|^2$ to the Hamiltonian density, where λ_{BF} is the fermion-boson coupling constant. Furthermore, all fermions occupy in the same spin state so that the short-range inter-fermion interactions do not contribute by virtue of the Pauli exclusion principle.

We are interested in the dynamics of the reduced system of bosons described by the functional

$$S = S_{BEC} + \text{Tr} \log \left[\hbar \partial_\tau - \frac{\hbar^2 \nabla^2}{2m_F} - \mu_F + \lambda_{BF} |\Psi_B|^2 \right], \quad (1)$$

where S_0 is the action of the bosons alone, $S_{BEC} = \int d\tau (\hbar \int d^3\mathbf{r} \dot{\Psi}_B \Psi_B^* - H_{BEC})$, and the second term is a contribution due to the interaction of bosons with fermions; μ_F is the chemical potential of the fermions. Here and throughout the paper we will be utilizing the imaginary time (Matsubara) representation, unless stated otherwise. An explicit evaluation of the second term is a challenging task. However, here we are interested in the dynamics of the slow breathing mode of the BEC Ψ_B^0 , which can be treated in the self-similar density

approximation. This dynamics describes the longitudinal expansions (and contractions) of the condensate. Finite size effects such as the appearance of a non-vanishing excitation energy (gap) can decouple this mode from other excitation modes. Hence, Ψ_B^0 peaks at small frequencies (ω) and small wavevectors (\mathbf{q}), giving a Ψ_B^0 that is a slowly varying function of spacial and temporal coordinates. In such case the $\text{Tr log}[\dots]$ in Eq. (1) can be evaluated within the Thomas-Fermi approximation. A straightforward zero-temperature calculation yields

$$\frac{\delta \text{Tr log}[\dots]}{\delta \Psi_B^*} = \frac{\lambda_{BF} k_F^3}{3\pi^2} \text{Re} \left[1 - \frac{\lambda_{BF} |\Psi_B(\mathbf{r})|^2}{\mu_F} \right]^{3/2} \Psi_B, \quad (2)$$

where k_F is the Fermi wavevector. Eq. (2) represents an additional term in the Gross-Pitaevskii (GP) equation, $\delta S_{GP}/\delta \Psi_B^* = 0$, resulting from interaction with fermions. In order to analyze the physical meaning of Eq. (2) let us expand it in powers of Ψ_B . The first nontrivial contribution is a term $-2\lambda' |\Psi_B|^2 \Psi_B$, $\lambda' = (\lambda_{BF}^2 k_F^3 / 4\pi^2 \mu_F)$, which corresponds to the attraction between bosons mediated by interaction with fermions. For nonzero, but small ω and \mathbf{q} there is an additional term (of the order of λ_{BF}^2) related to the dissipation of the condensate due to the Landau damping, as we discuss below. The next order yields $\eta |\Psi_B|^4 \Psi_B$, $\eta = (k_F^3 \lambda_{BF}^3 / 8\pi^2 \mu_F^2)$. Unlike the previous term this one is positive, and represents reduction in the effective boson-boson attraction due to depletion of fermions in the regions of high density of the bosons. The next order terms (in λ_{BF}) prove to be unimportant as can be verified directly from Eq. (2). Therefore we will replace the potential energy contribution in GP equation given by Eq. (2) by the two terms discussed above [12].

To analyze the dynamics of the slow (breathing) mode described by the Hamiltonian

$$H = \int d^3\mathbf{r} \left[\frac{\hbar^2}{2m_B} |\nabla \Psi_B|^2 + \frac{m_B \omega_T^2 \mathbf{r}^2}{2} |\Psi_B|^2 + \frac{1}{2} (\lambda_{BB} - \lambda') |\Psi_B|^4 + \frac{\eta}{6} |\Psi_B|^6 \right], \quad (3)$$

we apply the time dependent variational principle. Since we are interested in ground state properties of Eq. (5) we use a spherically symmetric Gaussian trial wavefunction

$$\Psi_B^0(\mathbf{r}) = \frac{N_B^{1/2}}{\pi^{3/4} (x R_0)^{3/2}} \exp \left[-\frac{\mathbf{r}^2}{2(x R_0)^2} \right], \quad (4)$$

parameterized by a dimensionless parameter x that characterizes the BEC's spatial width in units of the zero point motion amplitude $R_0 = (\hbar/2m_B \omega_T)^{1/2}$. Substitution of this wavefunction into Eq. (5) yields the following dependence of the ground-state energy E^0 on x :

$$E_0(x) = \frac{3N_B \hbar \omega_T}{2} \left(\frac{1}{x^2} + \frac{x^2}{4} - \frac{\alpha}{3x^3} + \frac{\beta}{6x^6} \right), \quad (5)$$

where $\alpha = N_B(\lambda' - \lambda_{BB})/[(2\pi)^{3/2} R_0^3 \hbar \omega_T]$ and $\beta = 4N_B^2 \eta / (3^{5/2} \pi^3 R_0^6 \hbar \omega_T)$. For positive but relatively small

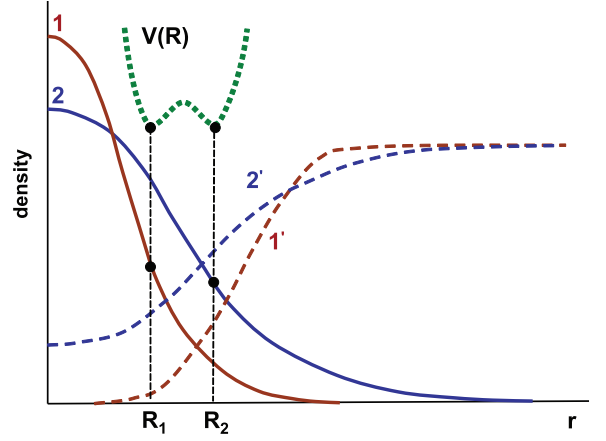


FIG. 1: Density profiles of the bosons (solid lines 1 and 2) and corresponding density profiles of the fermions (dashed lines 1' and 2') in two metastable states: (1) with fermions having zero density at the center of the trap ("separated phase") and (2) with nonzero density of fermions ("mixed phase"). The dotted line represents schematically an effective potential for the breathing mode of the bosons.

α , i.e., for $\alpha < \alpha_{cr} = 32(2/5)^{1/4}/15 \simeq 1.69$, E_0 may develop two competing minima, depending on the value of the β -parameter. The energy barrier separating the minima is caused by the same effect as the barrier appearing in the description of a BEC with attractive interactions: it arises due to the competition between the kinetic and the interaction energies, i.e., the first and the third terms in the right-hand side (rhs) of Eq. (5). In the absence of the last term in the rhs of Eq. (5) the state in this well would have been metastable - the energy would tend to $-\infty$ at $x \rightarrow 0$. The $1/x^6$ term stabilizes the system: for small x this term rapidly increases, giving rise to another minimum of $E_0(x)$, now due to the competition between the last two terms in the rhs of Eq. (5).

At certain values of α and β the two minima of $E_0(x)$ will have the same energy, and the ground state of the system becomes degenerate. Since our system is finite, this degeneracy will be lifted by the quantum tunneling transition between the two states. Such mechanism has been suggested to be the dominant decay process for condensates with attractive interactions between particles [5]. The tunneling corresponds to the low energy excitations of the breathing mode, described by the wavefunction in Eq. (4). It has been shown in [5] that by accounting for the superfluid motion of the condensate (which can be done by introducing a phase-factor $e^{i\phi}$ for the wavefunction in Eq. (4) and requiring the superfluid velocity $\mathbf{v}_s = (\hbar/m_B) \nabla \phi$ to satisfy the continuity equation) one obtains an effective action for the breathing mode of the condensate

$$S_0[x(\tau)] = \int d\tau \left[\frac{m_0 \dot{x}^2}{2} + E_0(x) \right], \quad (6)$$

where $E_0(x)$ is given by Eq. (5) and $m_0 = 3m_B N_B R_0^2/2$. Thus the dynamics of the ground state wavefunction of

the condensate is that of a quantum particle of mass m_0 moving in the potential $E_0(x)$.

A direct analysis of the Shrodinger equation corresponding to Eq. (6), however, is quite cumbersome since the two wells are generally quite asymmetric. Instead we choose an alternative route: we compute the ground state energy and obtain the tunneling rate by numerically solving the time-independent GP equation, $\delta H/\delta \Psi_B = E\Psi_B$, where H is given by Eq. (3). The latter approach also serves as an independent justification of the variational method and confirms that macroscopic quantum tunneling, QMT, is the mechanism that causes the transition between the two states of the condensate. Upon substitution $\Psi_B \sim \phi/r$, the time-independent GP equation can be cast in the form

$$\left[-\frac{\partial^2}{\partial x^2} + \frac{x^2}{4} - a\frac{\phi^2}{x^2} + \frac{b}{4}\frac{\phi^4}{x^4} \right] \phi = \mu\phi, \quad (7)$$

where the $\phi(x)$ -function is normalized to unity, $a = (\pi/2)^{1/2}\alpha$, $b = 3^{5/2}\pi\beta/16$, and $x = r/R_0$, $\mu = E/\hbar\omega_T$. We find the ground state numerically by replacing the rhs of Eq. (7) by $-\partial_\tau\phi$ and propagating ϕ in the imaginary time τ until it converges to the ground state ϕ_0 (or Ψ_B^0). We then evaluate the ground state energy according to Eq. (3) and present the results in Fig. 2(a) as a function of the b -parameter for different values of a . Fig. 2(b) shows the dispersion of the ground state width, $(1/N_B) \int d^3\mathbf{r} |\Psi_B^0(\mathbf{r})|^2 \mathbf{r}^2$, as a function of those same parameters. For $a < a_{cr} = 1.83$ the ground state energy and dispersion undergo a sharp crossover between the state with compressed and expanded BEC wavefunctions (corresponding to the phase separated and mixed states) as functions of b . Note that the value a_{cr} corresponds to the value of $\alpha_{cr} = 1.46$, which is quite close to the above critical value of 1.69 obtained from the variational approach. The dependence of ground state energy near the critical value of a is shown in the inset of Fig. 2(a). Clearly the ground state energy exhibits *avoided level crossing*, which is in accordance with the above conjecture (e.g., Eq. (6)) of macroscopic quantum tunneling between the two local energy minima.

The value of the tunneling matrix element Δ between two local “ground” states ϵ_1 and ϵ_2 can be deduced by fitting the calculated energy curves in Fig. 1 with the standard expression [13], $\epsilon = (\epsilon_1 + \epsilon_2)/2 - [(\epsilon_1 - \epsilon_2)^2/4 - \Delta^2]^{1/2}$ and assuming that in the vicinity of the point of crossover both ϵ_1 and ϵ_2 are linear functions of parameter b . For $a = 1.81$ one finds $\Delta \sim 10^{-4} \times \hbar\omega_T$, while for $a = 1.82$, $\Delta \sim 10^{-2} \times \hbar\omega_T$. Assuming that the ground state wavefunctions have Gaussian shape, $|\Psi_B|_{1(2)}^2 \sim R_{1(2)}^{-3} \exp(-\mathbf{r}^2/R_{1(2)}^2)$, from Fig. 2 one finds that $\bar{R} = (R_1 + R_2)/2 \simeq 0.85R_0$ for both $a = 1.81$ and $a = 1.82$, and $\delta R = |R_1 - R_2| \simeq 0.09R_0$ for $a = 1.81$ and $\delta R \simeq 0.03R_0$ for $a = 1.82$. For a typical value of the trapping frequency $\nu_T = 10^2 \text{ Hz}$ ($\omega_T = 2\pi\nu_T$), the two tunneling rates are $\Delta_{1.81}/\hbar = 10^{-2} \text{ s}^{-1}$ and $\Delta_{1.82}/\hbar = 10^2 \text{ s}^{-1}$. Since the value of R_0 for most trapped atomic BEC’s is of the order of a few microns,

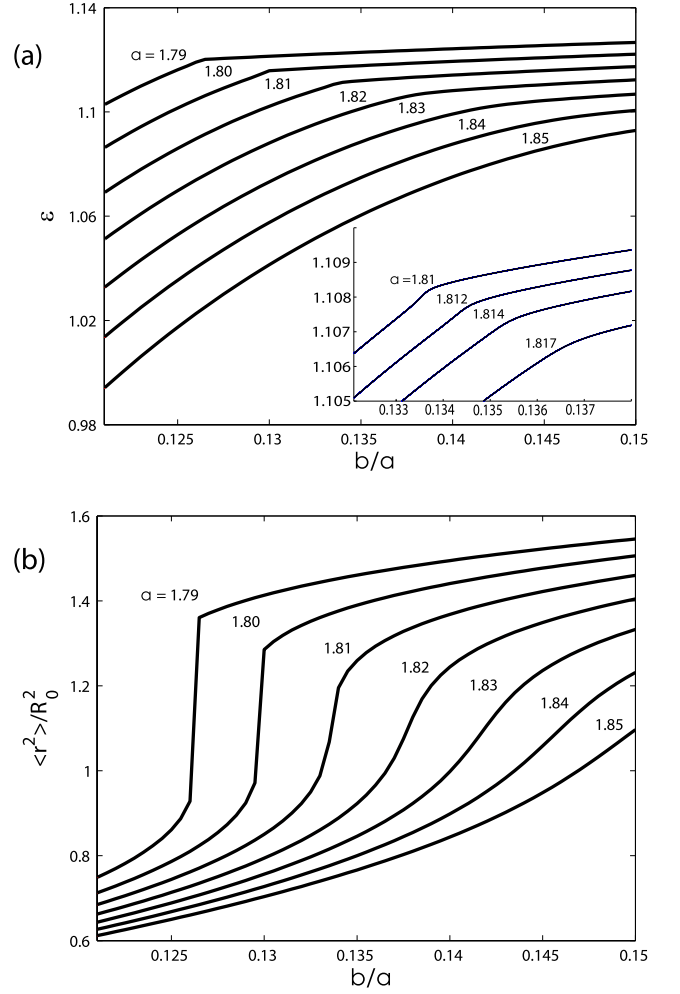


FIG. 2: (a) Dependence of the ground state energy of the BEC (per particle, in units of $\hbar\omega_T$) as a function of parameters a and b ; (b) Dispersion of the ground state spatial extent as a function of the same parameters.

the difference between the radii of the two condensate states δR is submicron. Such small variation may be difficult to observe in situ by optical means. However, the expansion process that takes place in time-of-flight measurements after the trap potential is shut off and the expanding atoms are observed, has successfully magnified small distance features in other experiments.

Role of dissipation: The above analysis determines the tunneling rate, but does not address the question whether the tunneling process is quantum coherent. Will the probability of the system to occupy one of the two macroscopic states oscillate in time as $\cos^2(\Delta t/\hbar)$? The fermions not only provide the BEC with the effective interaction, they also cause fluctuations which can destroy the macroscopic quantum coherence. To evaluate the effect of fluctuations, it is sufficient to consider the first non-vanishing frequency-dependent contribution into the effective action of the bosons coming from the perturba-

tive expansion of the $\text{Tr log}[\dots]$ term in Eq. (1):

$$-\frac{\lambda_{BF}^2}{2\hbar} \int \frac{d\omega}{2\pi} \int \frac{d^3\mathbf{q}}{(2\pi)^3} \chi_0(\mathbf{q}, \omega) |\rho_B(\omega, \mathbf{q})|^2. \quad (8)$$

Here $\rho_B(\mathbf{q}, \omega)$ is the Fourier transform of $\rho_B(\mathbf{r}, t)$ and χ_0 is the response function of the non-interacting fermions. In the small frequency domain $\chi_0 = (1/4\pi)[\hbar^2 k_F^3/(\pi\mu_F) + m_F^2|\omega|/(\hbar^2 q)]$. The frequency-independent part of χ_0 has already been incorporated in the effective interaction between bosons, i.e., $\lambda'|\Psi_B|^4$ term in Eq. (3). The second term in χ_0 is responsible for damping. To quantify its role we employ a two-state approximation in describing the tunneling dynamics. In this representation the tunneling is described by the Hamiltonian $H_{\text{tun}} = \Delta\hat{\sigma}_x$, where $\hat{\sigma}_x$ is a Pauli matrix with non-zero off-diagonal elements, and the position operator, i.e. the spatial width of the ground-state BEC wavefunction, is given by $\hat{R} = \bar{R} + (\delta R/2)\hat{\sigma}_z$, where $\hat{\sigma}_z$ is the diagonal Pauli matrix (with ± 1 along the diagonal). The dissipative part of the action for H_{tun} can be derived from Eq. (8) by substituting a Gaussian ansatz, $\rho_B(\mathbf{r}, t) = N_B/[\pi^{3/2}R^3(t)] \exp[-\mathbf{r}^2/R^2(t)]$, where $R(t) = \bar{R} + (\delta R/2)\sigma_z(t)$, $\sigma_z = \pm 1$, into Eq. (8). For $\delta R \ll \bar{R}$ one obtains

$$S_{\text{diss}} = \gamma\hbar \int d\tau d\tau' \sigma_z(\tau)\sigma_z(\tau')(\tau - \tau')^{-2}, \quad (9)$$

where $\gamma = N_B^2 \lambda_{BF}^2 m_F^2 \delta R^2 / [2(2\pi\hbar)^4 \bar{R}^4]$. Eq. (9), together with H_{tun} defined above, describes dissipative dynamics of a two-state system. Such dynamics has been extensively studied in connection with macroscopic quantum tunneling of a superconducting phase in Josephson junctions, and is known to depend critically on the value

the parameter γ . Specifically, for $\gamma > 1$ the two-state oscillation is always overdamped and at zero temperature it exhibits localization as a result of quantum fluctuations [14]. It is therefore instructive to evaluate γ for our situation. For estimates we consider an atomic mixture of ^{23}Na (bosons) and ^{40}K (fermions), which have natural scattering lengths $a_{BB} \simeq 1\text{ nm}$ ($\lambda_{BB} = 4\pi\hbar^2 a_{BB}/m_B$) and $a_{BF} \simeq 4\text{ nm}$ ($\lambda_{BF} = 2\pi\hbar^2 a_{BF}[(1/m_B) + (1/m_F)]$). For these data we obtain a critical value of $N_B^{cr} \simeq 12400$ (again for $\nu_T = 10^2\text{ Hz}$) and the fermion density $n_F^{cr} \simeq 7.4 \times 10^{15}\text{ cm}^{-3}$. Then, for $a = 1.81$ we obtain $\gamma_{1.81} \simeq 1.1$, which corresponds to the localized case (at $T = 0$), whereas for $a = 1.82$ one gets $\gamma_{1.82} \simeq 0.1$. In the high temperature limit (for $k_B T > \Delta$) the relaxation rate Γ can be expressed in terms of γ as $\hbar\Gamma = \pi\gamma k_B T$ [14], and therefore coherent (underdamped) oscillations can be observed for $T \ll \Delta_{1.82}/(\gamma_{1.82} k_B) = 0.5\text{ nK}$. The situation can be improved, however, if one utilizes a Feshbach resonance [4] to increase the a_{BF} scattering length. For example, for $a_{BF} = 80\text{ nm}$ one finds $N_B^{cr} \simeq 25$ and $n_F^{cr} \simeq 2.6 \times 10^{11}\text{ cm}^{-3}$, and $\gamma_{1.82} \simeq 2.5 \times 10^{-4}$. For such parameters coherent oscillations can be observed for $T \ll 0.2\text{ }\mu\text{K}$, which is easily observable. A low particle number also reduces the uncertainty of an atomic counting measurement that can be carried out in the time-of-flight procedure [15].

In summary we argue that a trapped boson-fermion mixture can exhibit MQT tunneling and coherent oscillations. Our studies indicate that MQT can be observed in ^{23}Na and ^{40}K atomic mixtures of sufficiently low temperatures.

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